Probing nuclear symmetry energy with the sub-threshold pion production

Zhang Fang

School of Nuclear Science and Technology, Lanzhou University, Lanzhou 730000, China

Liu Yang, Yong Gao-Chan and Zuo Wei

Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China

Within the framework of semiclassical Boltzmann-Uehling-Uhlenbeck (BUU) transport model, we investigated the effects of symmetry energy on the sub-threshold pion using the isospin MDI interaction with the stiff and soft symmetry energies in the central collision of ^{48}Ca + ^{48}Ca at the incident beam energies of 100, 150, 200, 250 and 300 MeV/nucleon, respectively. We find that the ratio of π^-/π^+ of sub-threshold charged pion production is greatly sensitive to the symmetry energy, particularly around 100 MeV/nucleon energies. Large sensitivity of sub-threshold charged pion production to nuclear symmetry energy may reduce uncertainties of probing nuclear symmetry energy via heavy-ion collision.

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The density dependence of the nuclear symmetry energy is not only important for nuclear physics, but also crucial for many astrophysical processes, such as the structure of neutron stars and the dynamical evolution of proto-neutron stars [1]. Though considerable progress has been made recently in determining the density dependence of the nuclear symmetry energy around the normal nuclear matter density from studying the isospin diffusion in heavy-ion reactions at intermediate energies [2–4], much more work is still needed to probe the high-density behavior of the nuclear symmetry energy. Fortunately, heavy-ion reactions, especially those induced by radioactive beams, provide a unique opportunity to constrain the EOS of asymmetric nuclear matter, and a number of such observables have been already identified in heavy-ion collisions induced by neutron-rich nuclei, such as the free neutron/proton ratio [5], the isospin fractionation [6, 7], the neutron-proton transverse differential flow [8], the neutron-proton correlation function [9], t/3He [10], the isospin diffusion [11], the proton differential elliptic flow [12]. Currently, to pin down the symmetry energy, the National Superconducting Cyclotron Laboratory (NSCL) at Michigan State University, the Geselschaft fuer Schwerionenforschung (GSI) at Darmstadt, the Rikagaku Kenkyusho (RIKEN, The Institute of Physical and Chemical Research) of Japan, as well as the Cooler Storage Ring (CSR) in Lanzhou, are planning to do related experiments. Some of related information can be find via Ref. [13].

Recently, pion production in heavy-ion collisions has attracted much attention in the nuclear physics community [14, 15, 19, 21]. One important reason for this is that pion production is connected with the high-density behavior of nuclear symmetry energy, especially around pion production threshold. The latter is crucial for understanding many interesting issues in both nuclear physics and astrophysics. And several hadronic transport models have quantitatively shown that π^-/π^+ ratio is indeed sensitive to the symmetry energy, especially around pion production threshold [19] and above pion

production threshold [24]. In the previous studies, pion production mainly from energetic heavy-ion collisions, in which pions are mainly produced above their threshold energy. What is the case of effects of nuclear symmetry energy on charged pion ratio via sub-threshold production? To answer this question, we studies the effects of nuclear symmetry energy on sub-threshold pion production. We select ^{48}Ca+^{48}Ca as the reaction system due to it large asymmetry. Based on the isospin-dependent Boltzmann-Uehling- Uhlenbeck (IBUU) transport model, we studied the effects of the symmetry energy in the central reaction ^{48}Ca+^{48}Ca and find that the ratio of π^-/π^+ of sub-threshold charged pion production, compared with above threshold case, is particularly sensitive to the symmetry energy.

Our present work is based on the semi-classical transport model IBUU04, in which nucleons, Δ and N^* resonances as well as pions and their isospin-dependent dynamics are included. We use the isospin-dependent in-medium nucleon-nucleon (NN) elastic cross sections from the scaling model according to nucleon effective masses. For the inelastic cross sections we use the experimental data from free space NN collisions since at higher incident beam energies the NN cross sections have no evident effects on the slope of neutron-proton differential flow. The total and differential cross sections for all other particles are taken either from experimental data or obtained by using the detailed balance formula. The isospin dependent phase-space distribution functions of the particles involved are solved by using the test-particle method numerically. The isospin-dependence of Pauli blockings for fermions is also considered. The isospin-dependence of Pauli blockings for fermions is also considered. More details can be found in Refs. [4, 5, 16–19]. In the present studies, the momentum-dependent single nucleon potential (MDI) adopted here is:

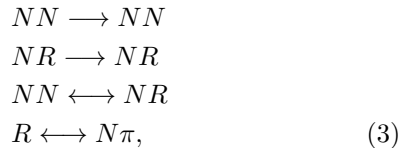$$U(\rho, \delta, \mathbf{p}, \tau) = A_u(x)\frac{\rho_{\tau'}}{\rho_0} + A_l(x)\frac{\rho_\tau}{\rho_0}$$

$$+B(\frac{\rho}{\rho_0})^\sigma(1 - x\delta^2) - 8x\tau\frac{B}{\sigma+1}\frac{\rho^{\sigma-1}}{\rho_0^\sigma}\delta\rho_{\tau'}$$

$$+\frac{2C_{\tau,\tau}}{\rho_0}\int d^3\mathbf{p}'\frac{f_\tau(\mathbf{r},\mathbf{p}')}{1+(\mathbf{p}-\mathbf{p}')^2/\Lambda^2}$$

$$+\frac{2C_{\tau,\tau'}}{\rho_0}\int d^3\mathbf{p}'\frac{f_{\tau'}(\mathbf{r},\mathbf{p}')}{1+(\mathbf{p}-\mathbf{p}')^2/\Lambda^2}. \quad (1)$$

In the above equation, $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$ is the isospin asymmetry parameter, $\rho = \rho_n + \rho_p$ is the baryon density and ρ_n, ρ_p are the neutron and proton densities, respectively. $\tau = 1/2(-1/2)$ for neutron (proton) and $\tau \neq \tau'$, $\sigma = 4/3$, $f_\tau(\mathbf{r},\mathbf{p})$ is the phase-space distribution function at coordinate $\mathbf{r}$ and momentum $\mathbf{p}$. The parameters $A_u(x), A_l(x), B, C_{\tau,\tau}, C_{\tau,\tau'}$ and Λ were set by reproducing the momentum-dependent potential $U(\rho, \delta, \mathbf{p}, \tau)$ predicted by the Gogny Hartree-Fock and/or the Brueckner-Hartree-Fock calculations. The momentum-dependence of the symmetry potential steams from the different interaction strength parameters $C_{\tau,\tau'}$ and $C_{\tau,\tau}$ for a nucleon of isospin τ interacting, respectively, with unlike and like nucleons in the background fields, more specifically, $C_{unlike} = -103.4 \ MeV$ while $C_{like} = -11.7 \ MeV$. The parameters $A_u(x)$and $A_l(x)$ depend on the x parameter according to $Au(x) = -95.98 - x\frac{2B}{\sigma+1}$ and $A_l(x) = -120.57 + x\frac{2B}{\sigma+1}$. The saturation properties of symmetric nuclear matter and the symmetry energy of about 32 MeV at normal nuclear matter density $\rho_0 = 0.16$ fm^{-3}. The incompressibility of symmetric nuclear matter at normal density is set to be 211 MeV. According to essentially all microscopic model calculations, the EOS for isospin asymmetric nuclear matter can be expressed as

$$E(\rho, \delta) = E(\rho, 0) + E_{\rm sym}(\rho)\delta^2 + \mathcal{O}(\delta^4), \quad (2)$$

where $E(\rho, 0)$ and $E_{\rm sym}(\rho)$ are the energy per nucleon of symmetric nuclear matter and nuclear symmetry energy, respectively. For a given value x, with the single particle potential $U(\rho, \delta, \mathbf{p}, \tau)$, one can readily calculate the symmetry energy $E_{\rm sym}(\rho)$ as a function of density.

The main reaction channels related to pion production and absorption are

$$NN \longrightarrow NN$$
$$NR \longrightarrow NR$$
$$NN \longleftrightarrow NR$$
$$R \longleftrightarrow N\pi, \quad (3)$$

where R denotes Δ or N^* resonances. In the present work, we use the isospin-dependent in-medium reduced NN elastic scattering cross section from the scaling model according to nucleon effective mass [19] to study the effect of symmetry energy on pion production. Assuming in-medium NN scattering transition matrix is the same as that in vacuum, the elastic NN scattering cross section in medium σ_{NN}^{medium} is reduced compared with their free-space value σ_{NN}^{free} by a factor of [20]

$$R_{medium}(\rho, \delta, \mathbf{p}) \equiv \sigma_{NN_{elastic}}^{medium}/\sigma_{NN_{elastic}}^{free}$$

$$= (\mu_{NN}^*/\mu_{NN})^2. \quad (4)$$

where μ_{NN} and μ_{NN}^* are the reduced masses of the colliding nucleon pair in free space and medium, respectively. For in-medium NN inelastic scattering cross section, even assuming in-medium $NN \rightarrow NR$ scattering transition matrix is the same as that in vacuum, the density of final states D_f' of NR is very hard to calculate due to the fact that the resonance's potential in matter is presently unknown. The in-medium NN inelastic scattering cross section is thus quite controversial. Because the purpose of present work is just study the effect of symmetry energy on pion production and charged pion ratio, to simplify the question, for the NN inelastic scattering cross section we use the free NN inelastic scattering cross section. The effective mass of nucleon in isospin asymmetric nuclear matter is

$$\frac{m_\tau^*}{m_\tau} = \left\{1 + \frac{m_\tau}{p}\frac{dU_\tau}{dp}\right\}^{-1}. \quad (5)$$

From the definition and Eq. (1), we can see that the effective mass depends not only on density and asymmetry of medium but also the momentum of nucleon.
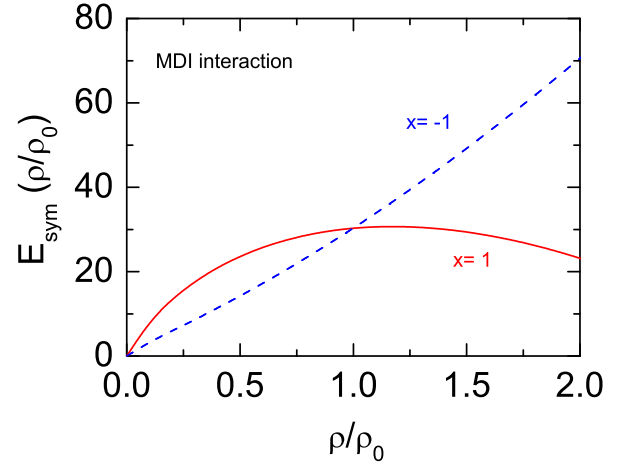

FIG. 1: (Color online) Density dependence of nuclear symmetry energy with parameters $x = 1, -1$, respectively.

Fig. 1 shows the density dependence of nuclear symmetry energy with parameter $x = 1, -1$, respectively. As discussed in the previous part, the single particle used has an x parameter, different specific x parameter denotes different density dependent symmetry energy. For the central reaction ^{48}Ca+^{48}Ca, the maximal density reached is about $1.5 \sim 2$ times saturation density as shown in Fig. 2. From Fig. 1, we can also see that the low density behaviors of nuclear symmetry energy separate from each other with different x parameters. At the saturation point there is a cross and then they separate
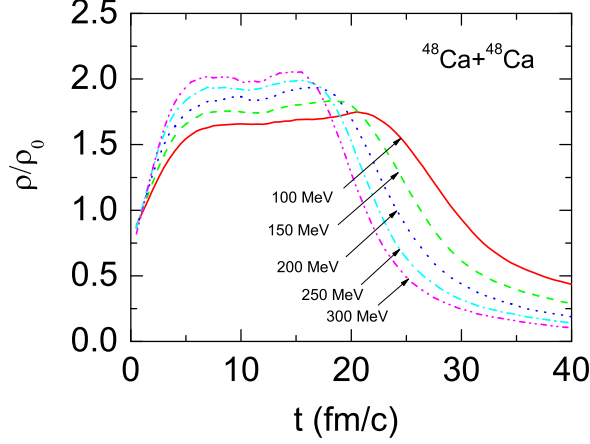
FIG. 2: (Color online) Maximal baryon density reached in the central reaction ^{48}Ca+^{48}Ca with different incident beam energies.

FIG. 3: (Color online) Kinetic energy distribution of the π^-/π^+ ratio using the MDI interaction with $x = 1$ and -1 for the central collision of ^{48}Ca + ^{48}Ca at the incident beam energies of 150 MeV/nucleon.

from each other again. At lower densities, the value of symmetry energy of $x = 0$ is lower than that of $x = 1$. But at high densities, the value of symmetry energy of $x = 0$ is higher than that of $x = 1$. From Fig. 2, we can see that the maximal baryon density reached in the central reaction ^{48}Ca+^{48}Ca increases with the incident beam energy. At 100 MeV/nucleon, the maximal baryon density reaches about 1.5 times saturation density and at 300 MeV/nucleon, the maximal baryon density reaches about 2 times saturation density. Therefore the ratio of π^-/π^+ of sub-threshold charged pion production still mainly reflects the high density behavior of nuclear symmetry energy.

Fig. 3 shows the kinetic energy distribution of the π^-/π^+ ratio using the MDI interaction with $x = 1$ and -1 for the central collision of ^{48}Ca + ^{48}Ca at the incident beam energies of 150 MeV/nucleon. It is seen that at pion kinetic energies of $30 \sim 120$ MeV, the π^-/π^+ ratio is very sensitive to the symmetry energy. The soft symmetry energy (x= 1) gives larger π^-/π^+ ratio whereas the stiff symmetry energy ($x = -1$) gives smaller π^-/π^+ ratio. This is consistent with the previous studies [14, 19, 21] for the neutron-rich reactions. And we can see that the sensitivity of π^-/π^+ ratio to the high density behavior of symmetry energy is quite large, about 80%, is quite sensitive to cases of pion production at higher incident beam energies [22]. The sub-threshold charged pion production is thus a very sensitive probe of the symmetry energy. Fig. 4 is the case at 300 MeV/nucleon incident beam energy. Again we clearly see that at pion kinetic energies of $30 \sim 120$ MeV, the π^-/π^+ ratio is sensitive to the symmetry energy. The soft symmetry energy ($x = 1$) gives larger π^-/π^+ ratio whereas the stiff symmetry energy ($x = -1$) gives smaller π^-/π^+ ratio. Sensitivity of
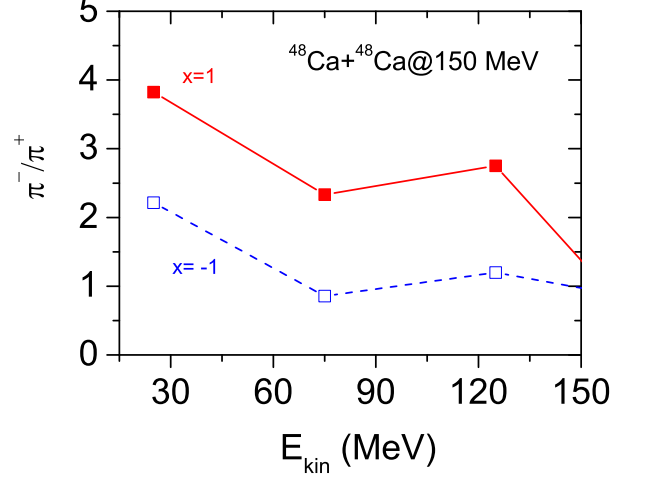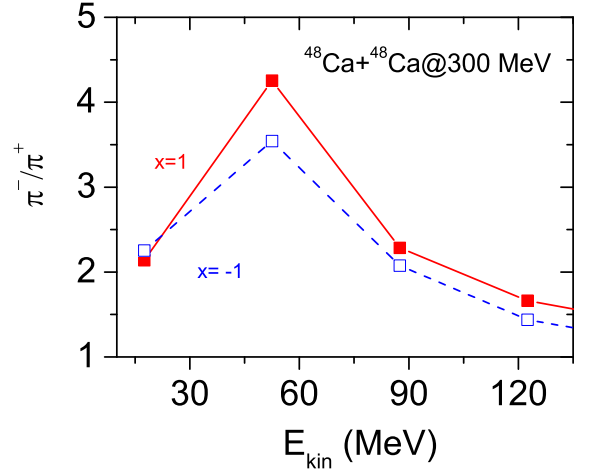

FIG. 4: (Color online) Kinetic energy distribution of the π^-/π^+ ratio using the MDI interaction with $x = 1$ and -1 for the central collision of ^{48}Ca + ^{48}Ca at the incident beam energies of 300 MeV/nucleon.

charged pion ratio to the symmetry energy at higher incident beam energy 300 MeV/nucleon is smaller than the case at lower incident beam energy 150 MeV/nucleon. In the heavy-ion collisions at intermediate energies, generally speaking, mean-field effect and collision effect compete each other. The effect of mean-field increases with the decreasement of incident beam energy. Thus at lower

incident beam energies effects of the symmetry energy (which is the isovector part of nuclear mean-field potential) are larger than that with higher incident beam energies. From Fig. 4, we can also see that there is an evident Coulomb peak [19], and around the Coulomb peak π^-/π^+ ratio is more sensitive to the symmetry energy. At lower incident beam energy 150 MeV/nucleon, however, there is no Coulomb peak at at pion kinetic energies of $30 \sim 120$ MeV.
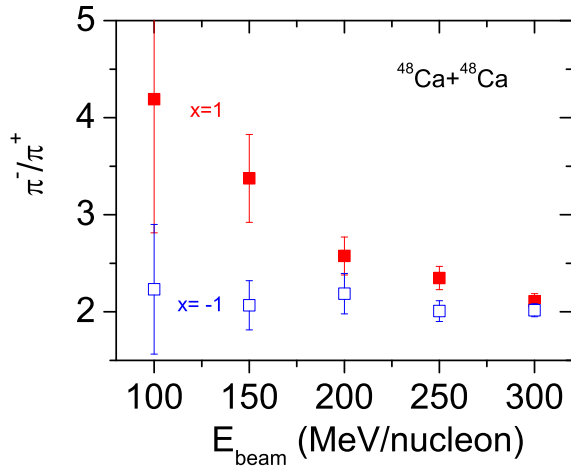

FIG. 5: (Color online) Excitation function of the π^-/π^+ ratio using the MDI interaction with $x = 1$ and -1 for the central collision of ^{48}Ca + ^{48}Ca at the incident beam energies of 100, 150, 200, 250 and 300 MeV/nucleon, respectively.

Fig. 5 shows the excitation function of the π^-/π^+ ratio using the MDI interaction with $x = 1$ and -1 for the central collision of ^{48}Ca + ^{48}Ca at the incident beam energies of 100, 150, 200, 250 and 300 MeV/nucleon, respectively. From Fig. 5 we can clearly see that sensitivity of charged pion ratio π^-/π^+ reaches the maximum at the lower incident beam energy 100 MeV/nucleon, about 100% sensitive to the symmetry energy. The trend of the sensitivity of the charged pion ratio π^-/π^+ to symmetry energy decreases with the incident beam energy is consistent with the previous studies [14]. Note here that at the incident beam energy region studied here, the soft symmetry energy always corresponds large value of π^-/π^+ ratio and the stiff symmetry energy corresponds relative small value of π^-/π^+ ratio [23]. However, in Ref. [24], Feng et al. systematically investigated the pion production in heavy-ion collisions in the region of below 1 AGeV energies by using the ImIQMD model. They found that the excitation functions of the π^-/π^+ ratio increases with the stiffness of the symmetry energy. This is inconsistent with our studies *qualitatively* and the reasons are needed to be clarified.

In summary, based on the semiclassical Boltzmann-Uehling-Uhlenbeck (BUU) transport model, we studied the effects of symmetry energy on the sub-threshold pion production in the central reaction ^{48}Ca+^{48}Ca. We find that the ratio of π^-/π^+ of sub-threshold charged pion production is particularly sensitive to the symmetry energy with decrease of incident beam energy of heavy-ion collision. The highly sensitive charged sub-threshold pion ratio to the symmetry energy may reduce the uncertainties of probing nuclear symmetry energy via heavy-ion collision.

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